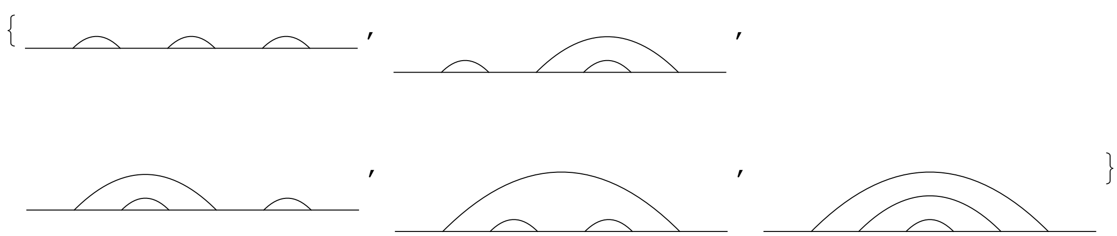
Pensieve header: Plotting the eigenvalues of the Temperley-Lieb inner product.

```
SetAttributes[p, Orderless];
CLP[{}] = {1};
CLP[l_List] := CLP[l] = Flatten[Table[
     Flatten[Outer[Times, CLP[l[[2 ;; k - 1]]], CLP[l[[k + 1 ;; Length[l]]]]]] *
      p[l[[1]], l[[k]]],
     {k, 2, Length[l], 2}
    ]];
DrawPairing[P_] := Graphics[{
   Line[{{0, 0}, {1 + Max[Flatten[List @@ P /. p → List]], 0}}],
   List @@ P /. p[i_, j_] ⧴ BezierCurve[{{i, 0}, {(i + j) / 2, (j - i) / 2}, {j, 0}}]
  }]
```

```
CLP[Range[8]]
```

```
{p[1, 2] p[3, 4] p[5, 6] p[7, 8], p[1, 2] p[3, 4] p[5, 8] p[6, 7],
 p[1, 2] p[3, 6] p[4, 5] p[7, 8], p[1, 2] p[3, 8] p[4, 5] p[6, 7],
 p[1, 2] p[3, 8] p[4, 7] p[5, 6], p[1, 4] p[2, 3] p[5, 6] p[7, 8],
 p[1, 4] p[2, 3] p[5, 8] p[6, 7], p[1, 6] p[2, 3] p[4, 5] p[7, 8],
 p[1, 6] p[2, 5] p[3, 4] p[7, 8], p[1, 8] p[2, 3] p[4, 5] p[6, 7],
 p[1, 8] p[2, 3] p[4, 7] p[5, 6], p[1, 8] p[2, 5] p[3, 4] p[6, 7],
 p[1, 8] p[2, 7] p[3, 4] p[5, 6], p[1, 8] p[2, 7] p[3, 6] p[4, 5]}
```

```
DrawPairing /@ CLP[Range[6]]
```



```
CLP[Range[8]] // Length
```

```
14
```

```
Hom[n_, m_] := CLP[Join[-Range[n], Range[m]]]
```

```
Hom[3, 5]
```

```
{p[-3, 1] p[-2, -1] p[2, 3] p[4, 5], p[-3, 1] p[-2, -1] p[2, 5] p[3, 4],
 p[-3, 3] p[-2, -1] p[1, 2] p[4, 5], p[-3, 5] p[-2, -1] p[1, 2] p[3, 4],
 p[-3, 5] p[-2, -1] p[1, 4] p[2, 3], p[-3, -2] p[-1, 1] p[2, 3] p[4, 5],
 p[-3, -2] p[-1, 1] p[2, 5] p[3, 4], p[-3, -2] p[-1, 3] p[1, 2] p[4, 5],
 p[-3, 1] p[-2, 2] p[-1, 3] p[4, 5], p[-3, -2] p[-1, 5] p[1, 2] p[3, 4],
 p[-3, -2] p[-1, 5] p[1, 4] p[2, 3], p[-3, 1] p[-2, 2] p[-1, 5] p[3, 4],
 p[-3, 1] p[-2, 4] p[-1, 5] p[2, 3], p[-3, 3] p[-2, 4] p[-1, 5] p[1, 2]}
```

```
Hom[5, 3]
```

```
{p[-5, 1] p[-4, -3] p[-2, -1] p[2, 3], p[-5, 3] p[-4, -3] p[-2, -1] p[1, 2],
 p[-5, -4] p[-3, 1] p[-2, -1] p[2, 3], p[-5, -4] p[-3, 3] p[-2, -1] p[1, 2],
 p[-5, 1] p[-4, 2] p[-3, 3] p[-2, -1], p[-5, 1] p[-4, -1] p[-3, -2] p[2, 3],
 p[-5, 3] p[-4, -1] p[-3, -2] p[1, 2], p[-5, -4] p[-3, -2] p[-1, 1] p[2, 3],
 p[-5, -2] p[-4, -3] p[-1, 1] p[2, 3], p[-5, -4] p[-3, -2] p[-1, 3] p[1, 2],
 p[-5, 1] p[-4, 2] p[-3, -2] p[-1, 3], p[-5, -2] p[-4, -3] p[-1, 3] p[1, 2],
 p[-5, 1] p[-4, -3] p[-2, 2] p[-1, 3], p[-5, -4] p[-3, 1] p[-2, 2] p[-1, 3]}
```

```
Comp[p1_, p2_] := Module[
  {t1, t2, t3},
  t1 = p1 * (p2 /. p[a_, b_] :> p[If[a < 0, -a, 100 + a], If[b < 0, -b, 100 + b]]);
  t2 = t1 //. p[a_, b_] p[b_, c_] :> p[a, c];
  t3 = t2 /. {p[a_, a_] → δ, p[a_, b_]^2 → δ};
  t3 /. p[a_, b_] :> p[If[a > 100, a - 100, a], If[b > 100, b - 100, b]]
 ]
```

```
Comp[p[-3, 3] p[-1, -2] p[1, 2] p[4, 5], p[-5, -4] p[-3, -2] p[-1, 1] p[2, 3]]
```

```
δ p[-3, 1] p[-2, -1] p[2, 3]
```

```
IP[p1_, p2_] := Module[
  {t1, t2},
  t2 = (p1 p2) //. p[a_, b_] p[b_, c_] :> p[a, c];
  t2 /. {p[a_, a_] → δ, p[a_, b_]^2 → δ}
 ]
```

```
IP[p[-3, 3] p[-1, -2] p[1, 2] p[4, 5], p[-5, -4] p[-3, -2] p[-1, 1] p[2, 3]]
```

```
δ p[-5, -4] p[4, 5]
```

```
M[n_] := Outer[IP, Hom[n, n], Hom[n, n]]
```

```
M[2] // MatrixForm
```

$$\begin{pmatrix} \delta^2 & \delta \\ \delta & \delta^2 \end{pmatrix}$$

```
M[3] // MatrixForm
```

$$\begin{pmatrix} \delta^3 & \delta^2 & \delta^2 & \delta & \delta^2 \\ \delta^2 & \delta^3 & \delta & \delta^2 & \delta \\ \delta^2 & \delta & \delta^3 & \delta^2 & \delta \\ \delta & \delta^2 & \delta^2 & \delta^3 & \delta^2 \\ \delta^2 & \delta & \delta & \delta^2 & \delta^3 \end{pmatrix}$$

```
M[4] // MatrixForm
```

$$\begin{pmatrix}
\delta^4 & \delta^3 & \delta^3 & \delta^2 & \delta^3 & \delta^3 & \delta^2 & \delta^2 & \delta^3 & \delta & \delta^2 & \delta^2 & \delta^3 & \delta^2 \\
\delta^3 & \delta^4 & \delta^2 & \delta^3 & \delta^2 & \delta^2 & \delta^3 & \delta & \delta^2 & \delta^2 & \delta & \delta^3 & \delta^2 & \delta \\
\delta^3 & \delta^2 & \delta^4 & \delta^3 & \delta^2 & \delta^2 & \delta & \delta^3 & \delta^2 & \delta^2 & \delta & \delta & \delta^2 & \delta^3 \\
\delta^2 & \delta^3 & \delta^3 & \delta^4 & \delta^3 & \delta & \delta^2 & \delta^2 & \delta & \delta^3 & \delta^2 & \delta^2 & \delta & \delta^2 \\
\delta^3 & \delta^2 & \delta^2 & \delta^3 & \delta^4 & \delta^2 & \delta & \delta & \delta^2 & \delta^2 & \delta^3 & \delta & \delta^2 & \delta \\
\delta^3 & \delta^2 & \delta^2 & \delta & \delta^2 & \delta^4 & \delta^3 & \delta^3 & \delta^2 & \delta^2 & \delta^3 & \delta & \delta^2 & \delta \\
\delta^2 & \delta^3 & \delta & \delta^2 & \delta & \delta^3 & \delta^4 & \delta^2 & \delta & \delta^3 & \delta^2 & \delta^2 & \delta & \delta^2 \\
\delta^2 & \delta & \delta^3 & \delta^2 & \delta & \delta^3 & \delta^2 & \delta^4 & \delta^3 & \delta^3 & \delta^2 & \delta^2 & \delta & \delta^2 \\
\delta^3 & \delta^2 & \delta^2 & \delta & \delta^2 & \delta^2 & \delta & \delta^3 & \delta^4 & \delta^2 & \delta & \delta^3 & \delta^2 & \delta \\
\delta & \delta^2 & \delta^2 & \delta^3 & \delta^2 & \delta^2 & \delta^3 & \delta^3 & \delta^2 & \delta^4 & \delta^3 & \delta^3 & \delta^2 & \delta^3 \\
\delta^2 & \delta & \delta & \delta^2 & \delta^3 & \delta^3 & \delta^2 & \delta^2 & \delta & \delta^3 & \delta^4 & \delta^2 & \delta^3 & \delta^2 \\
\delta^2 & \delta^3 & \delta & \delta^2 & \delta & \delta & \delta^2 & \delta^2 & \delta^3 & \delta^3 & \delta^2 & \delta^4 & \delta^3 & \delta^2 \\
\delta^3 & \delta^2 & \delta^2 & \delta & \delta^2 & \delta^2 & \delta & \delta & \delta^2 & \delta^2 & \delta^3 & \delta^3 & \delta^4 & \delta^3 \\
\delta^2 & \delta & \delta^3 & \delta^2 & \delta & \delta & \delta^2 & \delta^2 & \delta & \delta^3 & \delta^2 & \delta^2 & \delta^3 & \delta^4
\end{pmatrix}$$

```
M[5] // MatrixForm
```

$$\begin{pmatrix}
\delta^5 & \delta^4 & \delta^4 & \delta^3 & \delta^4 & \delta^4 & \delta^3 & \delta^3 & \delta^4 & \delta^2 & \delta^3 & \delta^3 & \delta^4 & \delta^3 & \delta^4 & \delta^3 & \delta^3 & \delta^2 & \delta^3 & \delta^3 & \delta^2 & \delta^4 & \delta^3 & \delta^2 & \delta^3 & \delta^3 & \delta^4 & \delta^3 \\
\delta^4 & \delta^5 & \delta^3 & \delta^4 & \delta^3 & \delta^3 & \delta^4 & \delta^2 & \delta^3 & \delta^3 & \delta^2 & \delta^4 & \delta^3 & \delta^2 & \delta^3 & \delta^4 & \delta^2 & \delta^3 & \delta^2 & \delta^2 & \delta^3 & \delta^3 & \delta^4 & \delta & \delta^2 & \delta^2 & \delta^3 & \delta^2 \\
\delta^4 & \delta^3 & \delta^5 & \delta^4 & \delta^3 & \delta^3 & \delta^2 & \delta^4 & \delta^3 & \delta^3 & \delta^2 & \delta^2 & \delta^3 & \delta^4 & \delta^3 & \delta^2 & \delta^4 & \delta^3 & \delta^2 & \delta^2 & \delta & \delta^3 & \delta^2 & \delta^3 & \delta^2 & \delta^4 & \delta^3 & \delta^2 \\
\delta^3 & \delta^4 & \delta^4 & \delta^5 & \delta^4 & \delta^2 & \delta^3 & \delta^3 & \delta^2 & \delta^4 & \delta^3 & \delta^3 & \delta^2 & \delta^3 & \delta^2 & \delta^3 & \delta^3 & \delta^4 & \delta^3 & \delta & \delta^2 & \delta^2 & \delta^3 & \delta^2 & \delta & \delta^3 & \delta^2 & \delta \\
\delta^4 & \delta^3 & \delta^3 & \delta^4 & \delta^5 & \delta^3 & \delta^2 & \delta^2 & \delta^3 & \delta^3 & \delta^4 & \delta^2 & \delta^3 & \delta^2 & \delta^3 & \delta^2 & \delta^2 & \delta^3 & \delta^4 & \delta^2 & \delta & \delta^3 & \delta^2 & \delta & \delta^2 & \delta^2 & \delta^3 & \delta^2 \\
\delta^4 & \delta^3 & \delta^3 & \delta^2 & \delta^3 & \delta^5 & \delta^4 & \delta^4 & \delta^3 & \delta^3 & \delta^4 & \delta^2 & \delta^3 & \delta^2 & \delta^3 & \delta^2 & \delta^2 & \delta & \delta^2 & \delta^4 & \delta^3 & \delta^3 & \delta^2 & \delta^3 & \delta^2 & \delta^2 & \delta^3 & \delta^4 \\
\delta^3 & \delta^4 & \delta^2 & \delta^3 & \delta^2 & \delta^4 & \delta^5 & \delta^3 & \delta^2 & \delta^4 & \delta^3 & \delta^3 & \delta^2 & \delta^3 & \delta^2 & \delta^3 & \delta & \delta^2 & \delta & \delta^3 & \delta^4 & \delta^2 & \delta^3 & \delta^2 & \delta & \delta & \delta^2 & \delta^3 \\
\delta^3 & \delta^2 & \delta^4 & \delta^3 & \delta^2 & \delta^4 & \delta^3 & \delta^5 & \delta^4 & \delta^4 & \delta^3 & \delta^3 & \delta^2 & \delta^3 & \delta^2 & \delta & \delta^3 & \delta^2 & \delta & \delta^3 & \delta^2 & \delta^2 & \delta & \delta^4 & \delta^3 & \delta^3 & \delta^2 & \delta^3 \\
\delta^4 & \delta^3 & \delta^3 & \delta^2 & \delta^3 & \delta^3 & \delta^2 & \delta^4 & \delta^5 & \delta^3 & \delta^2 & \delta^4 & \delta^3 & \delta^2 & \delta^3 & \delta^2 & \delta^2 & \delta & \delta^2 & \delta^2 & \delta & \delta^3 & \delta^2 & \delta^3 & \delta^4 & \delta^2 & \delta^3 & \delta^2 \\
\delta^2 & \delta^3 & \delta^3 & \delta^4 & \delta^3 & \delta^3 & \delta^4 & \delta^4 & \delta^3 & \delta^5 & \delta^4 & \delta^4 & \delta^3 & \delta^4 & \delta & \delta^2 & \delta^2 & \delta^3 & \delta^2 & \delta^2 & \delta^3 & \delta & \delta^2 & \delta^3 & \delta^2 & \delta^2 & \delta & \delta^2 \\
\delta^3 & \delta^2 & \delta^2 & \delta^3 & \delta^4 & \delta^4 & \delta^3 & \delta^3 & \delta^2 & \delta^4 & \delta^5 & \delta^3 & \delta^4 & \delta^3 & \delta^2 & \delta & \delta & \delta^2 & \delta^3 & \delta^3 & \delta^2 & \delta^2 & \delta & \delta^2 & \delta & \delta & \delta^2 & \delta^3 \\
\delta^3 & \delta^4 & \delta^2 & \delta^3 & \delta^2 & \delta^2 & \delta^3 & \delta^3 & \delta^4 & \delta^4 & \delta^3 & \delta^5 & \delta^4 & \delta^3 & \delta^2 & \delta^3 & \delta & \delta^2 & \delta & \delta & \delta^2 & \delta^2 & \delta^3 & \delta^2 & \delta^3 & \delta & \delta^2 & \delta \\
\delta^4 & \delta^3 & \delta^3 & \delta^2 & \delta^3 & \delta^3 & \delta^2 & \delta^2 & \delta^3 & \delta^3 & \delta^4 & \delta^4 & \delta^5 & \delta^4 & \delta^3 & \delta^2 & \delta^2 & \delta & \delta^2 & \delta^2 & \delta & \delta^3 & \delta^2 & \delta & \delta^2 & \delta^2 & \delta^3 & \delta^2 \\
\delta^3 & \delta^2 & \delta^4 & \delta^3 & \delta^2 & \delta^2 & \delta^3 & \delta^3 & \delta^2 & \delta^4 & \delta^3 & \delta^3 & \delta^4 & \delta^5 & \delta^2 & \delta & \delta^3 & \delta^2 & \delta & \delta & \delta^2 & \delta^2 & \delta & \delta^2 & \delta & \delta^3 & \delta^2 & \delta \\
\delta^4 & \delta^3 & \delta^3 & \delta^2 & \delta^3 & \delta^3 & \delta^2 & \delta^2 & \delta^3 & \delta & \delta^2 & \delta^2 & \delta^3 & \delta^2 & \delta^5 & \delta^4 & \delta^4 & \delta^3 & \delta^4 & \delta^4 & \delta^3 & \delta^3 & \delta^2 & \delta^3 & \delta^4 & \delta^2 & \delta^3 & \delta^2 \\
\delta^3 & \delta^4 & \delta^2 & \delta^3 & \delta^2 & \delta^2 & \delta^3 & \delta & \delta^2 & \delta^2 & \delta & \delta^3 & \delta^2 & \delta & \delta^4 & \delta^5 & \delta^3 & \delta^4 & \delta^3 & \delta^3 & \delta^4 & \delta^2 & \delta^3 & \delta^2 & \delta^3 & \delta & \delta^2 & \delta \\
\delta^3 & \delta^2 & \delta^4 & \delta^3 & \delta^2 & \delta^2 & \delta & \delta^3 & \delta^2 & \delta^2 & \delta & \delta & \delta^2 & \delta^3 & \delta^4 & \delta^3 & \delta^5 & \delta^4 & \delta^3 & \delta^3 & \delta^2 & \delta^2 & \delta & \delta^4 & \delta^3 & \delta^3 & \delta^2 & \delta^3 \\
\delta^2 & \delta^3 & \delta^3 & \delta^4 & \delta^3 & \delta & \delta^2 & \delta^2 & \delta & \delta^3 & \delta^2 & \delta^2 & \delta & \delta^2 & \delta^3 & \delta^4 & \delta^4 & \delta^5 & \delta^4 & \delta^2 & \delta^3 & \delta & \delta^2 & \delta^3 & \delta^2 & \delta^2 & \delta & \delta^2 \\
\delta^3 & \delta^2 & \delta^2 & \delta^3 & \delta^4 & \delta^2 & \delta & \delta & \delta^2 & \delta^2 & \delta^3 & \delta & \delta^2 & \delta & \delta^4 & \delta^3 & \delta^3 & \delta^4 & \delta^5 & \delta^3 & \delta^2 & \delta^2 & \delta & \delta^2 & \delta^3 & \delta & \delta^2 & \delta \\
\delta^3 & \delta^2 & \delta^2 & \delta & \delta^2 & \delta^4 & \delta^3 & \delta^3 & \delta^2 & \delta^2 & \delta^3 & \delta & \delta^2 & \delta & \delta^4 & \delta^3 & \delta^3 & \delta^2 & \delta^3 & \delta^5 & \delta^4 & \delta^4 & \delta^3 & \delta^4 & \delta^3 & \delta^3 & \delta^2 & \delta^3 \\
\delta^2 & \delta^3 & \delta & \delta^2 & \delta & \delta^3 & \delta^4 & \delta^2 & \delta & \delta^3 & \delta^2 & \delta^2 & \delta & \delta^2 & \delta^3 & \delta^4 & \delta^2 & \delta^3 & \delta^2 & \delta^4 & \delta^5 & \delta^3 & \delta^4 & \delta^3 & \delta^2 & \delta^2 & \delta & \delta^2 \\
\delta^4 & \delta^3 & \delta^3 & \delta^2 & \delta^3 & \delta^3 & \delta^2 & \delta^2 & \delta^3 & \delta & \delta^2 & \delta^2 & \delta^3 & \delta^2 & \delta^3 & \delta^2 & \delta^2 & \delta & \delta^2 & \delta^4 & \delta^3 & \delta^5 & \delta^4 & \delta^3 & \delta^2 & \delta^4 & \delta^3 & \delta^2 \\
\delta^3 & \delta^4 & \delta^2 & \delta^3 & \delta^2 & \delta^2 & \delta^3 & \delta & \delta^2 & \delta^2 & \delta & \delta^3 & \delta^2 & \delta & \delta^2 & \delta^3 & \delta & \delta^2 & \delta & \delta^3 & \delta^4 & \delta^4 & \delta^5 & \delta^2 & \delta & \delta^3 & \delta^2 & \delta \\
\delta^2 & \delta & \delta^3 & \delta^2 & \delta & \delta^3 & \delta^2 & \delta^4 & \delta^3 & \delta^3 & \delta^2 & \delta^2 & \delta & \delta^2 & \delta^3 & \delta^2 & \delta^4 & \delta^3 & \delta^2 & \delta^4 & \delta^3 & \delta^3 & \delta^2 & \delta^5 & \delta^4 & \delta^4 & \delta^3 & \delta^4 \\
\delta^3 & \delta^2 & \delta^2 & \delta & \delta^2 & \delta^2 & \delta & \delta^3 & \delta^4 & \delta^2 & \delta & \delta^3 & \delta^2 & \delta & \delta^4 & \delta^3 & \delta^3 & \delta^2 & \delta^3 & \delta^3 & \delta^2 & \delta^2 & \delta & \delta^4 & \delta^5 & \delta^3 & \delta^4 & \delta^3 \\
\delta^3 & \delta^2 & \delta^4 & \delta^3 & \delta^2 & \delta^2 & \delta & \delta^3 & \delta^2 & \delta^2 & \delta & \delta & \delta^2 & \delta^3 & \delta^2 & \delta & \delta^3 & \delta^2 & \delta & \delta^3 & \delta^2 & \delta^4 & \delta^3 & \delta^4 & \delta^3 & \delta^5 & \delta^4 & \delta^3 \\
\delta^4 & \delta^3 & \delta^3 & \delta^2 & \delta^3 & \delta^3 & \delta^2 & \delta^2 & \delta^3 & \delta & \delta^2 & \delta^2 & \delta^3 & \delta^2 & \delta^3 & \delta^2 & \delta^2 & \delta & \delta^2 & \delta^2 & \delta & \delta^3 & \delta^2 & \delta^3 & \delta^4 & \delta^4 & \delta^5 & \delta^4 \\
\delta^3 & \delta^2 & \delta^2 & \delta & \delta^2 & \delta^4 & \delta^3 & \delta^3 & \delta^2 & \delta^2 & \delta^3 & \delta & \delta^2 & \delta & \delta^2 & \delta & \delta^3 & \delta^2 & \delta & \delta^3 & \delta^2 & \delta^2 & \delta & \delta^4 & \delta^3 & \delta^3 & \delta^4 & \delta^5 \\
\delta & \delta^2 & \delta^2 & \delta^3 & \delta^2 & \delta^2 & \delta^3 & \delta^3 & \delta^2 & \delta^4 & \delta^3 & \delta^3 & \delta^2 & \delta^3 & \delta^2 & \delta^3 & \delta^3 & \delta^4 & \delta^3 & \delta^3 & \delta^4 & \delta^2 & \delta^3 & \delta^4 & \delta^3 & \delta^3 & \delta^2 & \delta^3 \\
\delta^2 & \delta & \delta & \delta^2 & \delta^3 & \delta^3 & \delta^2 & \delta^2 & \delta & \delta^3 & \delta^4 & \delta^2 & \delta^3 & \delta^2 & \delta^3 & \delta^2 & \delta^2 & \delta^3 & \delta^4 & \delta^4 & \delta^3 & \delta^3 & \delta^2 & \delta^3 & \delta^2 & \delta^2 & \delta & \delta^2 \\
\delta^2 & \delta^3 & \delta & \delta^2 & \delta & \delta & \delta^2 & \delta^2 & \delta^3 & \delta^3 & \delta^2 & \delta^4 & \delta^3 & \delta^2 & \delta^3 & \delta^4 & \delta^2 & \delta^3 & \delta^2 & \delta^2 & \delta^3 & \delta & \delta^2 & \delta^3 & \delta^4 & \delta^2 & \delta^3 & \delta^2 \\
\delta^3 & \delta^2 & \delta^2 & \delta & \delta^2 & \delta^2 & \delta & \delta & \delta^2 & \delta^2 & \delta^3 & \delta^3 & \delta^4 & \delta^3 & \delta^4 & \delta^3 & \delta^3 & \delta^2 & \delta^3 & \delta^3 & \delta^2 & \delta^2 & \delta & \delta^2 & \delta^3 & \delta & \delta^2 & \delta \\
\delta^2 & \delta & \delta^3 & \delta^2 & \delta & \delta & \delta^2 & \delta^2 & \delta & \delta^3 & \delta^2 & \delta^2 & \delta^3 & \delta^4 & \delta^3 & \delta^2 & \delta^4 & \delta^3 & \delta^2 & \delta^2 & \delta^3 & \delta & \delta^2 & \delta^3 & \delta^2 & \delta^2 & \delta & \delta^2 \\
\delta^2 & \delta^3 & \delta^3 & \delta^4 & \delta^3 & \delta & \delta^2 & \delta^2 & \delta & \delta^3 & \delta^2 & \delta^2 & \delta & \delta^2 & \delta & \delta^2 & \delta^2 & \delta^3 & \delta^2 & \delta^2 & \delta^3 & \delta^3 & \delta^4 & \delta^3 & \delta^2 & \delta^4 & \delta^3 & \delta^2 \\
\delta^3 & \delta^2 & \delta^2 & \delta^3 & \delta^4 & \delta^2 & \delta & \delta & \delta^2 & \delta^2 & \delta^3 & \delta & \delta^2 & \delta & \delta^2 & \delta & \delta & \delta^2 & \delta^3 & \delta^3 & \delta^2 & \delta^4 & \delta^3 & \delta^2 & \delta & \delta^3 & \delta^2 & \delta \\
\delta^3 & \delta^4 & \delta^2 & \delta^3 & \delta^2 & \delta^2 & \delta^3 & \delta & \delta^2 & \delta^2 & \delta & \delta^3 & \delta^2 & \delta & \delta^2 & \delta^3 & \delta & \delta^2 & \delta & \delta & \delta^2 & \delta^2 & \delta^3 & \delta^2 & \delta^3 & \delta^3 & \delta^4 & \delta^3 \\
\delta^2 & \delta^3 & \delta & \delta^2 & \delta & \delta^3 & \delta^4 & \delta^2 & \delta & \delta^3 & \delta^2 & \delta^2 & \delta & \delta^2 & \delta & \delta^2 & \delta^2 & \delta^3 & \delta^2 & \delta^2 & \delta^3 & \delta & \delta^2 & \delta^3 & \delta^2 & \delta^2 & \delta^3 & \delta^4 \\
\delta^4 & \delta^3 & \delta^3 & \delta^2 & \delta^3 & \delta^3 & \delta^2 & \delta^2 & \delta^3 & \delta & \delta^2 & \delta^2 & \delta^3 & \delta^2 & \delta^3 & \delta^2 & \delta^2 & \delta & \delta^2 & \delta^2 & \delta & \delta^3 & \delta^2 & \delta & \delta^2 & \delta^2 & \delta^3 & \delta^2 \\
\delta^3 & \delta^2 & \delta^4 & \delta^3 & \delta^2 & \delta^2 & \delta & \delta^3 & \delta^2 & \delta^2 & \delta & \delta & \delta^2 & \delta^3 & \delta^2 & \delta & \delta^3 & \delta^2 & \delta & \delta & \delta^2 & \delta^2 & \delta^3 & \delta^2 & \delta & \delta^3 & \delta^2 & \delta \\
\delta^3 & \delta^2 & \delta^2 & \delta & \delta^2 & \delta^4 & \delta^3 & \delta^3 & \delta^2 & \delta^2 & \delta^3 & \delta & \delta^2 & \delta & \delta^2 & \delta & \delta & \delta^2 & \delta^3 & \delta^3 & \delta^2 & \delta^2 & \delta & \delta^2 & \delta & \delta & \delta^2 & \delta^3 \\
\delta^2 & \delta & \delta^3 & \delta^2 & \delta & \delta^3 & \delta^2 & \delta^4 & \delta^3 & \delta^3 & \delta^2 & \delta^2 & \delta & \delta^2 & \delta & \delta^2 & \delta^2 & \delta^3 & \delta^2 & \delta^2 & \delta^3 & \delta & \delta^2 & \delta^3 & \delta^2 & \delta^2 & \delta & \delta^2 \\
\delta^3 & \delta^2 & \delta^2 & \delta & \delta^2 & \delta^2 & \delta & \delta^3 & \delta^4 & \delta^2 & \delta & \delta^3 & \delta^2 & \delta & \delta^2 & \delta^3 & \delta & \delta^2 & \delta & \delta & \delta^2 & \delta^2 & \delta & \delta^2 & \delta^3 & \delta & \delta^2 & \delta
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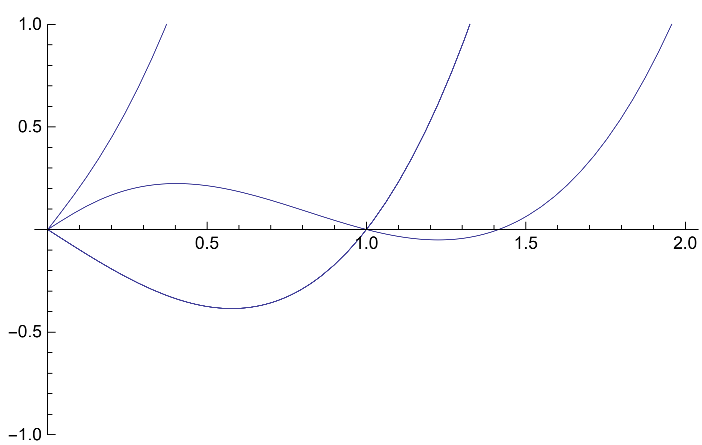

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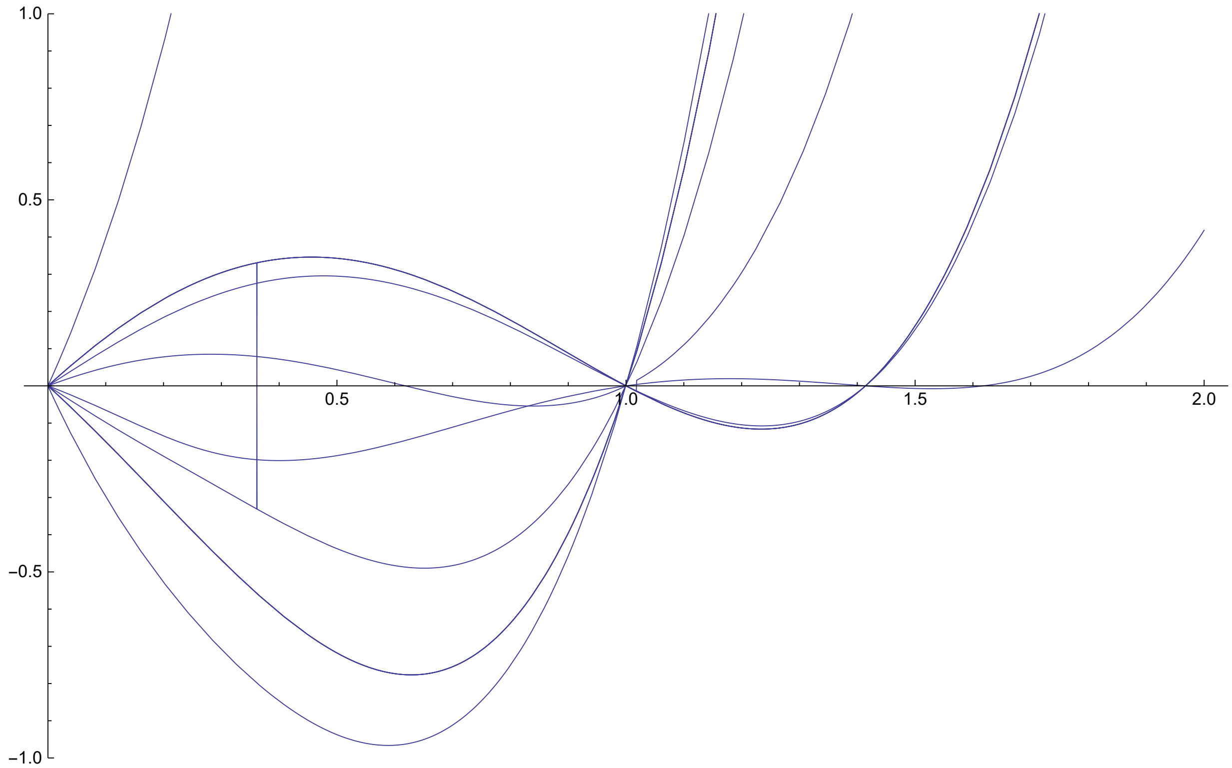


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